CHOICE OF OPTIMAL PARAMETERS OF RADIOISOTOPIC DEVICES USED IN THERMOPHYSICAL STUDIES

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Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 2, pp. 258-262, 1965

The choice of the basic parameters of γ-ray devices is discussed in relation to the problems of building heat physics.

The most rational criterion for choosing the optimal parameters of radioisotopic devices and the corresponding measuring conditions is the quantity of information obtained as a result of the experiment. The quantity of information is related to the concept of entropy [1], the value of which, in contrast to its use in thermophysical problems as a measure of indeterminacy, serves to characterize the problem. The entropy H is defined as:

$$H = a \sum_{i=1}^{n} P_i \log_2 P_i , \tag{1}$$

where P_i is the probability of the i-th possible solution of the problem. In particular, when all possible solutions of the problem are equiprobable, (1) becomes

$$H = \log_2 n. \tag{2}$$

In this case, the entropy has a maximum value, i.e., the indeterminacy of the solutions of the problem is greatest. If the probability distribution is continuous, then (1) may be written

$$H = - \int_{x_1}^{x_2} P(x) \log_2 P(x)\, dx, \tag{3}$$

where P(x) is the probability density of x. For the complete solution of a specific problem, we need to obtain the information G in a quantity numerically equal to the entropy H. Hence, in its most general form, the entropy will characterize numerically the possible accuracy needed for solving the problem, irrespective of the method used in its solution.

If the problem consists in the determination of a physical quantity ξ, varying from ξ_1 through ξ_2 with the same error $\Delta\xi$ over the whole interval $\xi_2 - \xi_1$, then, by (2), its characteristic entropy

$$H = - \log_2 \frac{\xi_2 - \xi_1}{2\Delta\xi} . \tag{4}$$

In this case, the quantity of information is conveniently characterized by the number of gradations [2]

$$g_\xi = \frac{\xi_2 - \xi_1}{2\Delta\xi} . \tag{5}$$

For a constant relative error $\pm\delta$ over the interval of variation, the number of gradations is written

$$g_\xi = \frac{1}{2\delta} \ln \frac{\xi_2}{\xi_1} . \tag{6}$$

Similarly, the number of gradations g_i of the detected intensity may be found, and this must satisfy the condition:

$$g_i \geqslant g_\xi . \tag{7}$$

Below are shown some possible applications of the above to the choice of required parameters of radioisotopic devices.

To choose the optimal parameters it is first necessary to estimate the expected measuring error. For radiometric measurements, the basic sources of error are: a) statistical fluctuations of the radiation – the statistical error; b) instability of the apparatus; c) effect of background and its variation in time; d) nonlinearity of the relation between the intensity of radiation I_{rad} and the detectable value I_d.

The statistical error can, in principle, be reduced to any value by the choice of corresponding source activity and measuring time. It is convenient in practice to make the statistical error of the same order of magnitude as the total error due to all other sources, since it may be expensive to reduce the statistical error – by decreasing the output or increasing the radiation hazard.

The instability of the measuring equipment depends on the setup, measuring conditions, and method of detection. Modern radiometric apparatus, which measures the intensity of radiation by counting pulses in gas discharge and scintillation detectors working in the integral regime at room temperature, gives an accuracy of 0.3-1% for single measurements. The background effect, which in the course of a working day changes by 5-10%, may be less than 1% if the lowest measured intensity is 10 times greater than the background.

The nonlinearity of the measuring channel is determined by the resolving time τ and also by the dead time of the counter and counting equipment. The usual correction for this error is

$$I_{\mathrm{rad}} = I_{\mathrm{r}}/(1 - \tau_{\mathrm{r}} I_{\mathrm{r}}), \tag{8}$$

in which τ_r is determined experimentally, for example by the two-source method [3]. The need to introduce a correction for nonlinearity raises the level of random errors, since the relative value of these errors is proportional to the nonlinearity and equal to from 1-10% of it (due to the inaccuracy in the determination of τ_r and triple coincidences).

This restricts the upper limit of the count rate by requiring a nonlinearity of 5-10% for an accuracy of 1-3%.

Thus, the total error of a single measurement may be less than 1-2% only in particularly favorable conditions. Considering that to calculate density or moisture content requires a minimum of three measurements, including calibrating, the mean total measuring error of a single determination may be estimated at 3%. The statistical error, the recommended value for which is not more than 1%, comprises a small fraction of the total error. In the majority of cases it is therefore permissible to assume the relative error constant over the range of variation of the parameter to be determined, and to use (6) to calculate the number of gradations, while estimating the sensitivity of the method as:

$$k = \left(\frac{\Delta I}{I}\right) \Big/ \left(\frac{\Delta \xi}{\xi}\right). \tag{9}$$

When the relation between the parameter to be determined and the measured value is close to linear, the sensitivity of the method may be estimated by means of an integral characteristic – the coefficient of relative differentiation

$$k_d = I_2/I_1. \tag{10}$$

Let us consider the choice of maximum permissible source energy for radiographic methods starting from the required accuracy of values of ρd and the possible accuracy of absorption measurements. From (6) and (7) the following approximate relation may be obtained:

$$\mu_{\mathrm{eff}} \geqslant \frac{\delta_i}{\delta_\xi} (\rho d)_{\mathrm{av}}^{-1}. \tag{11}$$

Let us calculate from (11) the maximum energy for narrow-beam radiography as a function of ρd for a ratio of the measuring accuracy δ_i to the permissible error δ_ξ in the determination of ρd $\delta_i/\delta_\xi = 3$ (i.e., for an accuracy of the value determined of 1%). For ρd = 5, 10, 25, 50, 75, 100, the energies in MeV are respectively: for water 0.02; 0.03; 0.17; 1.15; 2.5; 4.0; and for concrete 0.03; 0.05; 0.19; 1.0; 2.5; 4.0. For wide-beam radiography, analogous data may be obtained using the approximation $k = \exp(-\mu_{\mathrm{eff}} \rho d_{\mathrm{av}})$ for given $(\rho d)_{\mathrm{av}}$, the values of μ_{eff} being calculated or derived experimentally.

Other things being equal, a reduction in the energy of the source used increases the absorption coefficient, which in its turn increases the sensitivity. Moreover, in some types of γ-ray detector (scintillation counters) the sensitivity increases with decreasing energy of the γ-quanta detected. From this point of view, it is profitable to use low-energy sources. Furthermore, a decrease in source energy enhances the effect of variation of the chemical composition of the object investigated and of local inhomogeneities on the results of the measurements. The increased absorption for reduced source energy lowers the statistical accuracy of the measurements, so that for large thicknesses a substantial increase in source activity becomes necessary. In the γ-ray scattering method, reduced source energy results in a decrease in depth of measurement.

Source activity is calculated for the optimum count rate; as shown above, the minimum count rate must be at least one order greater than the background, and the maximum must lie within the limits of permissible nonlinearity of the measuring channel. The product of the count rate and measuring time must correspond to the calculated statistical detection error (for a permissible error of 1% not less than $I_r t_r$ = 10 000 counts).

The base measurement (distance between source and detector) in radiographic depends on the dimension of the object in the direction of radiography and the dimensions of the collimator, and sometimes on the arrangement of the experimental setup in the vicinity of the object investigated. If the experiment permits the thickness of the object to be fixed arbitrarily, it is more profitable to use a harder source, and the criterion for the choice of thickness is the necessary sensitivity of the measurements in accordance with (11).

TABLE

Sensitivity as a function of base measurement for scattering method

Weight by volume of object (n/m^3)	Sensitivity for values of base (mm)					
	160	210	260	310	360	410
14000	0.87	0.47	0.24	0.14	0.07	0.05
12000	0.17	0.57	0.30	0.18	0.10	0.06
10000	0.96	0.61	0.33	0.21	0.13	0.08
8000	0.96	0.63	0.35	0.24	0.15	0.10

In the scattering method there is a rather complex relation between base measurement, sensitivity, and depth of measurement. For measurements in the region of low weight by volume (up to 1000 kg/m^3) the pre-inversion region with a low base (up to 80 mm) is used and for large weight by volume, the post-inversion region with a high base (up to 500 mm). In this case, the base is usually determined by the maximum sensitivity and, more rarely, by the depth of measurement. The thickness of the object investigated must be greater than the depth of measurement. The dependence of the sensitivity on the base for the scattering method is shown in the table for several weights by volume (post-inversion region). Figure 1 shows the dependence of the depth of measurement on the base.

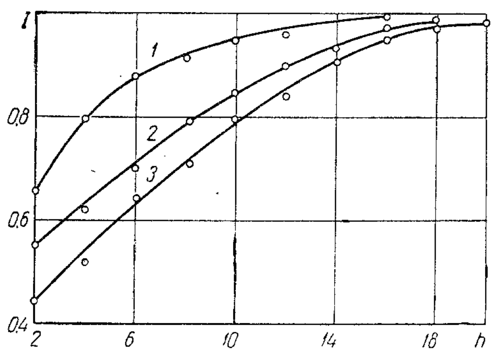

Fig. 1. Radiation intensity I as a function of depth of measurement h for different bases: 1 – base = 300 mm; 2 – 400; 3 – 500.

To increase the sensitivity and reduce the resolving time, it is useful to employ several counters connected in parallel, combined in a single cassette, in place of a single counter with a large cathode area. The spectral sensitivity of the counters is calculated for the detected spectrum, which depends on the method of measurement, the chemical composition of the material investigated, and the mean source energy. Thus, for example, in the narrow-beam method (with collimator), it is expedient to use a detector sensitive to high energies, such as a gas discharge counter with a copper or steel cathode. In the scattering method, on the other hand, detectors with increased sensitivity in the region of soft radiation are used (e.g., a gas discharge counter with a tungsten cathode). For wide-beam measurements it is most suitable to use a detector sensitive to the hard part of the spectrum, when the functional relation between detected radiation and measured parameter formally approximates the narrow-beam law.

Working with integrating circuits may introduce an additional error, on account of the non-correspondence of the time constant τ of the integrating system and the rate of change of the parameter investigated (detection time), which has a special value for measurements on objects moving relative to source and detector.

To obtain an error no greater than 1%, the detection time must be not less than 5τ seconds, and the lowest count rate to ensure the necessary statistical accuracy is

$$I_r \geqslant 3\cdot 10^5/\tau\delta^2 \text{ counts/min}, \tag{12}$$

where δ(%) is the mean square relative error due to fluctuations.

As an illustration, let us consider the application of the above principles to the choice of the optimum parameters of a setup to determine correct to 1% moisture content by layers of a lightweight concrete, with a weight by volume of about 800-1000 kg/m^3. The moisture content varies from 0-40%, i.e., the solution of the problem, according to (5), requires g_g = 20 gradations. We assume that the accuracy of measurement of the absorption coefficient I/I_0 is 3%. The desired accuracy, in accordance with (11), is reached for $\mu_{eff} \simeq 0.2$. The energy of a Tu^{170} source corresponds to this coefficient. The permissible minimum count rate is 10 000 counts/min (taking the time for a single determination of the radiation intensity to be 1 minute). We estimate the maximum count rate at 110 000 counts/min, assuming a possible density range of, in this case, 600 kg/m^3. For such a count rate, a nonlinearity of 5% is assured if the resolving time of the equipment is less than 30 μsec. We therefore use a scintillation and conversion arrangement. A collimated counter with a 35 × 25 NaI (Tl) phosphor has a background of about 1000 counts/min. In order to guarantee the required count rate for radiography of the sample, a source activity of 10 mg equivalent of Ra is selected.

When working with narrow-beam radiography, it is convenient to use a semi-logarithmic plot $\rho d = -1/\mu \ln K$. To construct the calibration curve it is sufficient to measure K for one standard sample with known ρd. I_0 is usually determined by direct irradiation of the detector. This is extremely undesirable, since measurement of a high intensity may introduce a considerable error on account of the nonlinearity of the measuring channel and fatigue in the photomultiplier. I_i/I_{st} may be measured in place of I_i/I_0, where I_{st} is determined for a standard sample with known $\mu\rho d$.

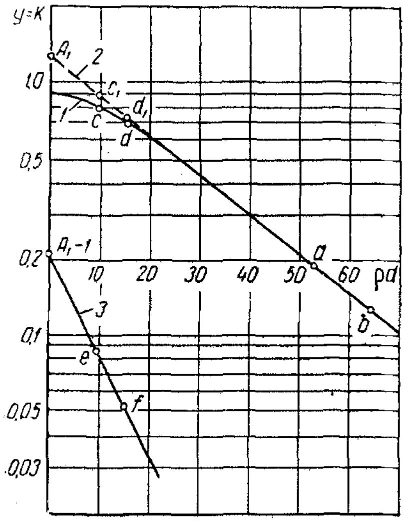

Fig. 2. Construction of graph for wide-beam measurements with a limited number of experimental points.

A wide-beam device is usually calibrated at several points lying within the probable measuring interval. To obtain the complete plot of $K = f(\rho d)$ with a limited number of standard samples, the following graphical procedure may be used. It is known that for source energies of 0.3-3.0 MeV and detectors insensitive to energies below 0.1 MeV the relation has the form $K = A_1 \exp(-\mu_1 \cdot \rho d) - (A_1 - 1)\exp(-\mu_2 \rho d)$.

From the results of two or three measurements (points a, b on Fig. 2) at maximal values of ρd, when the second term is practically zero, a straight line 2 may be projected to intersect with the y-axis. Measurements at low ρd give points c and d. The auxiliary exponent 3 is drawn from the ordinate $(A_1 - 1)$ through the two ordinates e and f, equal to the difference between the experimental points c and d and the points c_1 and d_1 on exponent 2. The difference of exponents 2 and 3 is the required function $K = f(\rho d)$. This method requires the minimum number of experimental points and at the same time gives a check on the quality of the calibration measurements.

In calibrating a device using scattered radiation, it should be borne in mind that over a wide range of weights by volume for instruments with a large base (post-inversion) the relation $I = f(\rho)$ is represented by a straight line in the coordinates $\ln I - \rho$ and for instruments with a small base (pre-inversion) by a straight line in linear coordinates. This permits the use of two or three calibration samples with extreme values of the weight by volume.

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13 May 1964

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